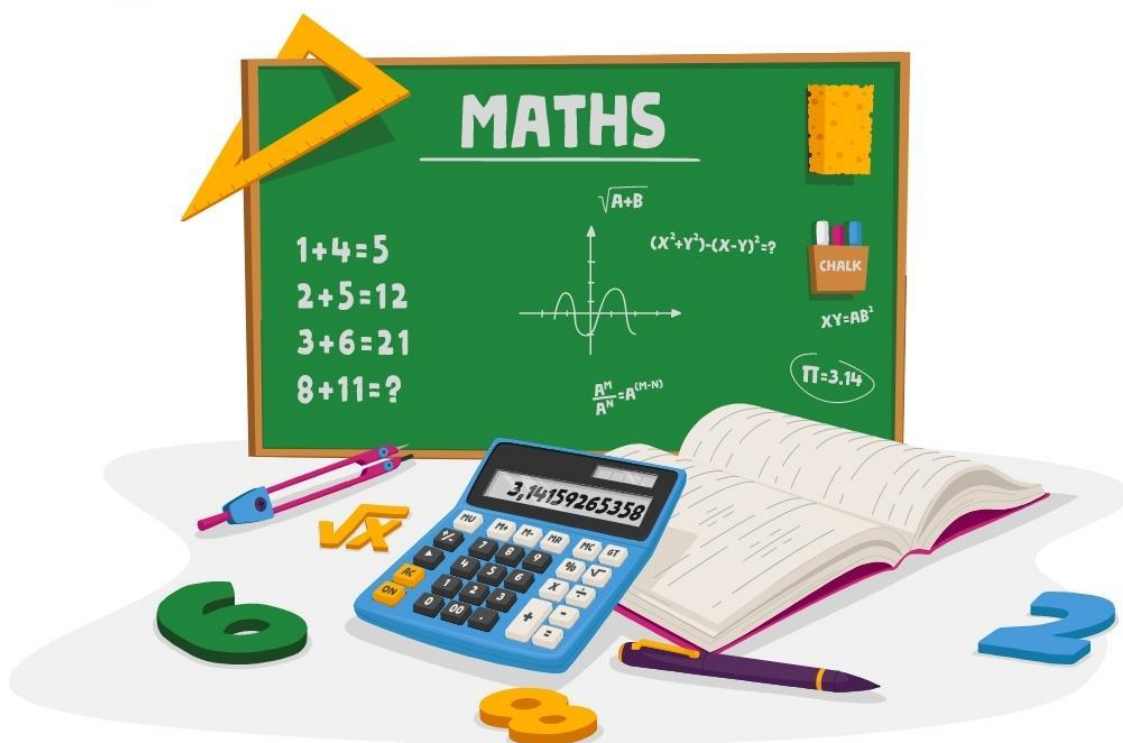




IB Maths AI HL Paper 2 Question Bank



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1. The family of curves $y = f(x) = (ax^3 + bx^2 + cx + d)^n$, where a, b, c, d , and n are constants, is given.

a) For $n = 2$, determine the inflection point of the function $y = f(x)$.

For $n = 2$, the function becomes $y = (ax^3 + bx^2 + cx + d)^2$. To find the inflection point, we need to find the second derivative of the function, which is:

$$f''(x) = 6a^2x + 2b$$

Now we need to find the x values where the second derivative is equal to zero.

$$6a^2x + 2b = 0$$

$$x = -b/(3a^2)$$

b) Next, Consider the curve, $y^2 = 2x^3 + x$

(i) Find the derivative of $y^2 = 2x^3 + x$

$$y^2 = 2x^3 + x$$

$$y = \sqrt{(2x^3 + x)}$$

Next, we can take the derivative of this expression using the chain rule. The chain rule states that if we have a function $u(x) = f(g(x))$, then the derivative of $u(x)$ with respect to x is given by:

$$du/dx = df/dg * dg/dx$$

In this case, we have $u(x) = \sqrt{(2x^3 + x)}$ and we have to find the derivative of this function with respect to x

$$du/dx = (1/2)(2x^3 + x)^{-1/2}(6x^2 + 1) = (3x(2x^2 + 1))/(2\sqrt{(2x^3 + x)})$$

So the derivative of $y = \sqrt{(2x^3 + x)}$ with respect to x is $(3x(2x^2 + 1))/(2\sqrt{(2x^3 + x)})$

(ii) Hence, find the local minima or maxima of $y^2 = 2x^3 + x$

To find the local maxima or minima of $y^2 = 2x^3 + x$, we need to find the critical points of the function $y = \sqrt{(2x^3 + x)}$.

A critical point of a function is a point where the derivative of the function is equal to zero or does not exist.

First, we need to find the derivative of $y = \sqrt{(2x^3 + x)}$ with respect to x :

$$y' = (3x(2x^2 + 1))/(2\sqrt{(2x^3 + x)})$$

Next, we need to set y' equal to zero and solve for x :

$$(3x(2x^2 + 1))/(2\sqrt{(2x^3 + x)}) = 0$$

$$3x(2x^2 + 1) = 0$$

$$x = 0$$

Now, we need to check if $x = 0$ is a local maximum or minimum. To do this, we need to find the second derivative of $y = \sqrt{(2x^3 + x)}$ with respect to x :

$$y'' = (6x(2x^2 + 1) - 3(2x^2 + 1)(2x))/(2\sqrt{(2x^3 + x)})^2$$

We need to evaluate y'' at $x = 0$, we get

$$y''(0) = -3/2, \text{ which is negative and that means that } x = 0 \text{ is a local minimum.}$$

So the local minima of $y^2 = 2x^3 + x$ is $(0, 0)$

2. An amount of \$ 10 000 is invested at an annual interest rate of 12%.

(a) Find the value of the investment after 5 years

(i) if the interest rate is compounded yearly;

$$FV = 10000(1 + 12/100)^n$$

$$n = 5, 17623.42$$

(ii) if the interest rate is compounded half-yearly;

$$n = 10, 17908.48$$

(iii) if the interest rate is compounded quarterly;

$$n = 20, 18061.11$$

(iv) if the interest rate is compounded monthly.

$$n = 5 \times 12, 18166.97$$

(b) The value of the investment will exceed \$ 20 000 after n full years. Calculate the minimum value of n

(i) if the interest rate is compounded yearly;

$$20000 = 10000(1 + 12/100)^n$$

Solve by GDC

$$n = 6.11, \text{ hence } n = 7$$

(ii) if the interest rate is compounded monthly.

$$20000 = 10000(1 + 12/100)^{12n}$$

Solve by GDC

$$n = 5.81, \text{ hence } n = 6$$

3. In the triangle PQR, PR = 5 cm, QR = 4 cm and PQ = 6 cm. Calculate

a) The size of PQR

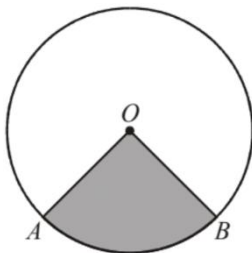
$$5^2 = 4^2 + 6^2 - 2(4)(6)(\cos Q)$$

$$PQR = 55.8^\circ$$

b) The area of the triangle PQR

$$\frac{1}{2} (4)(6)(\sin 55.8) = 9.92 \text{ cm}^2$$

4. O is the centre of the circle which has a radius of 5.4 cm.



Area of the shaded sector OAB is 21.6 cm^2 . Find the length of the minor arc AB

$$\frac{1}{2}(5.4)^2\theta = 21.6$$

$$\theta = 1.481 \text{ radians}$$

$$AB = r\theta = 5.4 \times 1.481 = 8 \text{ cm}$$

5. Consider the geometric sequence 10, 5, 2.5, 1.25, ...

(a) Write down the first term u_1 and the common ratio r .

$$u_1 = 10, r = 0.5$$

(b) Find the 10th term of the sequence.

$$0.0195$$

(c) Find the sum of the first 10 terms.

$$19.98$$

(d) Express the general term u_n in terms of n .

$$10 \times 0.5^{(n-1)}$$

(e) Hence find the value of n given that $u_n = 0.03125$

$$n = 6$$

6. An amount of \$ 10 000 is invested at an annual interest rate of 12%.

(a) Find the value of the investment after 5 years

(i) if the interest rate is compounded yearly;

$$FV = 10000(1 + 12/100)^n$$

$$n = 5, 17623.42$$

(ii) if the interest rate is compounded half-yearly;

$$n = 10, 17908.48$$

(iii) if the interest rate is compounded quarterly;

$$n = 20, 18061.11$$

(iv) if the interest rate is compounded monthly.

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(b) The value of the investment will exceed \$ 20 000 after n full years. Calculate the minimum value of n

(i) if the interest rate is compounded yearly;

$$20000 = 10000(1 + 12/100)^n$$

$$n = 6.11, \text{ hence } n = 7$$

(ii) if the interest rate is compounded monthly.

$$20000 = 10000(1 + 12/100)^{12n}$$

$n = 5.81$, hence $n = 6$

7. Let $f(x) = e^x - x^2$ and $g(x) = x^3 - 3x^2 + 2x + 1$

a) Find the derivative of $f(x)$ using the chain rule and the product rule.

$$f'(x) = e^x - 2x$$

b) Find the derivative of $g(x)$ using the power rule

$$g'(x) = 3x^2 - 6x + 2$$

c) Find the point(s) of the intersection of the graphs of $y = f(x)$ and $y = g(x)$ and state whether they are minima or maxima.

The point(s) of intersection can be found by solving the equation $f(x) = g(x)$, which gives $x = 0.5$ and $x = -0.5$. Since the second derivative of $f(x)$ at those points is positive, it follows that the points of intersection are minima.

d) Find the second derivative of $f(x)$ and express it in terms of x .

$$f''(x) = e^x - 2$$

e) Using the information from parts 1-4, determine the concavity of the graph of $y = f(x)$ and state the point(s) of inflection.

The second derivative of $f(x)$ is positive for $x < -0.5$ and $x > 0.5$, it follows that $f(x)$ is concave up on those intervals, and therefore the points of inflection are $x = -0.5$ and $x = 0.5$.

8. The weight X of a particular animal is normally distributed with $\mu = 200\text{kg}$ and $\sigma = 15\text{kg}$. An animal of this population is overweight if it has a weight greater than 230 kg

(a) Find the probability that an animal is overweight.

$$P(X > 230) = 0.02275$$

(b) We select 2 animals from this population. Find the probabilities that

(i) both animals are overweight

$$(0.02275)^2 = 0.000518$$

(ii) only one animal is overweight.

$$0.02275 * (1 - 0.02275) * 2 = 0.0445$$

(c) We select 7 animals from this population. Find the probability that exactly two of them are overweight.

Y follows $B(n, p)$ with $n = 7$, $p = 0.02275$

$$P(Y = 2) = 0.00969$$

10. A factory produces mobile components, where the probability of a component being defective is 0.05. The factory produces 1000 components per day.

a) Use the binomial distribution to find the probability that exactly 50 components are defective in a day.

$$P(X = 50) = (1000 \text{ C } 50) * (0.05)^{50} * (1-0.05)^{950}$$
$$p(X = 50) = 0.09$$

b) Use the binomial distribution to find the probability that less than 50 components are defective in a day.

$$P(X < 50) = P(X = 0) + P(X = 1) + P(X = 2) + \dots + P(X = 49)$$

c) Use the Poisson distribution to find the probability that exactly 50 components are defective in a day.

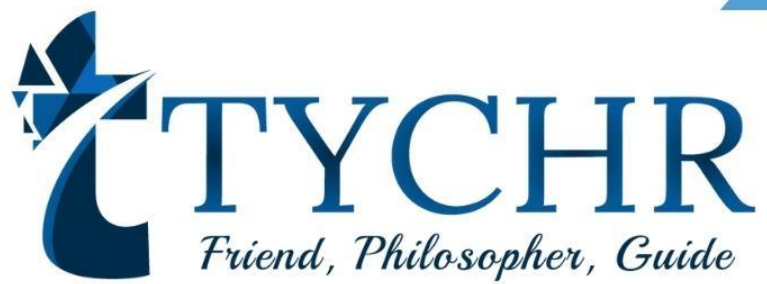
$$P(X = 50) = (e^{-5} * 5^{50}) / 50!$$

d) Compare the results obtained in parts 1 and 3 and explain the conditions under which the Poisson distribution can be used to approximate the binomial distribution.

Poisson distribution can be used as an approximation of the binomial distribution when the number of trials is large and the probability of success is small

e) Find the expected value and standard deviation of the number of defective components produced in a day.

$$E(X) = n * p = 1000 * 0.05 = 50 \text{ and } \sigma = \sqrt{np(1-p)} = \sqrt{(1000 * 0.05 * 0.95)} = 4.72$$



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